

TRANSFORMSReview of Laplace Transform

- The behaviour of the n/w can be written in equation by using KVL, KCL, n/w Theorems etc.
- These equ<sup>n</sup>s are always of integro-differential type.
- It has been realized that the method of solving the differential equ<sup>n</sup> in time domain is complicated. But using the Laplace Transform method, it becomes simple to obtain the solu<sup>n</sup> of differential equ<sup>n</sup>.
- The procedure for solving differential equ<sup>n</sup> in the time domain by using Laplace Transform method can be shown below.

Advantages of Laplace Transform Method :-

- It gives complete solu<sup>n</sup>.
- Initial conditions are automatically considered in the transformed equ<sup>n</sup>.
- Much less time is involved in solving differential equ<sup>n</sup>.
- It gives systematic and routine solu<sup>n</sup> for differential equ<sup>n</sup>.

## Defination of Laplace Transform. :-

Let  $f(t)$  be a function of time which is zero for  $t < 0$  and which is arbitrary defined for  $t > 0$ , subject to some conditions. Then the Laplace Transform of the function  $f(t)$ , denoted by  $F(s)$  is defined as

$$L[f(t)] = F(s) = \int_0^{\infty} f(t) \cdot e^{-st} \cdot dt.$$

## Laplace Transform for standard inputs :-

$$1. V(t) \rightarrow \text{unit step function} \longrightarrow \frac{1}{s}$$

$$2. V(t-T) \rightarrow \text{unit step function delayed by } T \longrightarrow \frac{e^{-sT}}{s}$$

$$3. \delta(t) \rightarrow \text{unit impulse} \longrightarrow 1$$

$$4. e^{at} \rightarrow (\text{exponential}) \longrightarrow \frac{1}{s-a}$$

$$5. e^{-at} \rightarrow (\text{exponential}) \longrightarrow \frac{1}{s+a}$$

$$6. \sin \omega t \rightarrow \text{sine function} \longrightarrow \frac{\omega}{s^2 + \omega^2}$$

$$7. \cos \omega t \rightarrow \text{cascine function} \longrightarrow \frac{s}{s^2 + \omega^2}$$

$$8. t^n (n=1, 2, 3, \dots) \rightarrow \text{ramp function} \longrightarrow \frac{n!}{s^{n+1}}$$

$$9. t \rightarrow \text{unit ramp function} \longrightarrow \frac{1}{s^2}$$

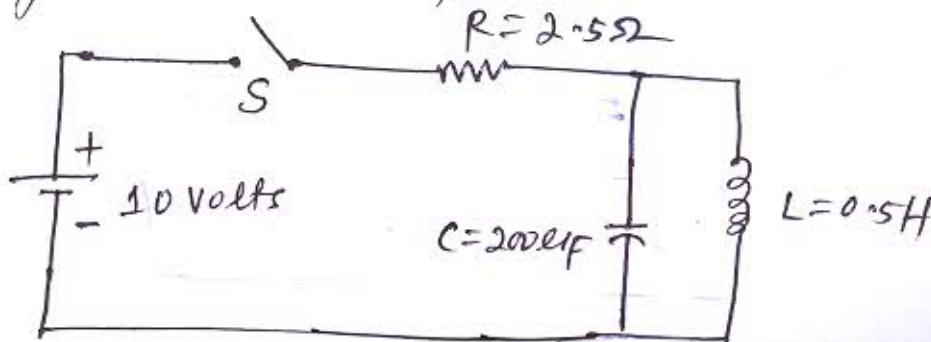
# Analyses of electrical circuits using Laplace.

Transform for standard inputs :-

Example - (1) on the o/p shown in figure, the switch  $S'$  is closed and steady state attained. At  $t=0$ , the switch is opened.

(a) Determine the current through the inductor.

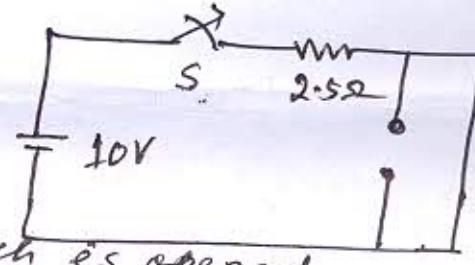
(b) Voltage across the capacitor at  $t=0.5$  second.



Solution

$$I(0) = \frac{10}{2.5} = 4A$$

Because at steady state inductor act as a short ckt and capacitor act as an open ckt. At  $t=0$ , switch is opened



$$L \frac{dI}{dt} + \frac{1}{C} \int I dt = 0$$

Taking Laplace Transform of above equ<sup>n</sup>.

$$sL I(s) - L I(0) + \frac{1}{Cs} I(s) = 0$$

$$\Rightarrow I(s) \left[ LS + \frac{1}{Cs} \right] = L I(0)$$

$$\Rightarrow I(s) \left[ s \times 0.5 + \frac{1}{200 \times 10^{-6} s} \right] = 0.5 \times 4 = 2$$

$$\Rightarrow I(s) = \frac{4s}{s^2 + 10^4}$$

$$\Rightarrow I(t) = 4 \cos 100t \rightarrow \text{current through the inductor.}$$

(b) Voltage across the capacitor at  $t = 0.5$  second

$$V_C = \frac{1}{C} \int_0^{0.5} 4 \cos 100t \cdot dt$$

$$= \frac{4}{2\pi \times 10^{-6} \times 100} \sin(100 \times 0.5)$$

$$= 153.2 \text{ volt}$$

Hence.

$$\boxed{V_C = 153.2 \text{ volt}}$$

CONVOLUTION INTEGRAL :

→ Convolution of two real functions corresponds to multiplication of their respective functions.

→ If  $L[f_1(t)] = F_1(s)$  and  $L[f_2(t)] = F_2(s)$   
convolution is defined by

$$L[f_1(t) * f_2(t)] = F_1(s) \cdot F_2(s) \quad \text{--- (1)}$$

→ The two functions  $f_1(t)$  and  $f_2(t)$  are multiplied in such a manner that one is continuously moving with time  $\tau$  (say) relative to the other.

i.e.  $f_1(t) * f_2(t) = \int_0^t f_1(t-\tau) \cdot f_2(\tau) \cdot d\tau \quad \text{--- (2)}$

→ The statement of the mathematical expression given in expression (1) is called Convolution Theorem.

Let  $L[f_1(t) * f_2(t)] = F(s)$

$$\Rightarrow F(s) = \int_0^\infty [f_1(t) * f_2(t)] e^{-st} \cdot dt$$

$$\Rightarrow F(s) = \int_{t=0}^\infty \left[ \int_{z=0}^t f_1(t-\tau) \cdot f_2(\tau) \cdot d\tau \right] e^{-st} \cdot dt$$

(3)

## INVERSE LAPLACE TRANSFORM

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→ The inverse Laplace Transform is the transformation of a Laplace transform into a function of time.

→ If  $L[f(t)] = F(s)$  then  $f(t)$  is the inverse Laplace Transform of  $F(s)$ .

$$L^{-1}[F(s)] = f(t)$$

Example - (1) Determine I.L.T for  $F(s) = \frac{s+1}{s(s+2)}$

Solu<sup>n</sup> given  $F(s) = \frac{s+1}{s(s+2)}$

$$= \frac{A}{s} + \frac{B}{s+2}$$

$$A = s \times F(s) \Big|_{s=0} = s \times \frac{s+1}{s(s+2)} \Big|_{s=0}$$

$$= \frac{1}{2}$$

$$B = (s+2) \times F(s) \Big|_{s=-2} = (s+2) \times \frac{s+1}{s(s+2)} \Big|_{s=-2}$$

$$= \frac{1}{2}$$

$$\text{Hence } F(s) = \frac{1}{2} \cdot \frac{1}{s} + \frac{1}{2} \cdot \frac{1}{(s+2)}$$

Taking inverse L.T of above function for unit step function.

$$f(t) = \underbrace{\frac{1}{2} + \frac{1}{2} e^{-2t}}_{\rightarrow 0} \quad t > 0$$

Example-(2) calculate S.L.T. for given function.

$$F(s) = \frac{2s+1}{(s+1)(s+2)(s+3)}$$

Solu<sup>n</sup>  $F(s) = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s+3}$

$$A = (s+1) \times F(s) \Big|_{s=-1}$$

$$= (s+1) \times \frac{2s+1}{(s+1)(s+2)(s+3)} \Big|_{s=-1} = -\frac{1}{2}$$

$$B = (s+2) \times F(s) \Big|_{s=-2}$$

$$= (s+2) \times \frac{2s+1}{(s+1)(s+2)(s+3)} \Big|_{s=-2} = 3$$

$$C = (s+3) \times F(s) \Big|_{s=-3}$$

$$= (s+3) \times \frac{2s+1}{(s+1)(s+2)(s+3)} \Big|_{s=-3} = -\frac{5}{2}$$

Hence.  $F(s) = \frac{(-\frac{1}{2})}{s+1} + \frac{3}{s+2} + \frac{(-5/2)}{s+3}$ .

We take I.L.T. of above equ<sup>n</sup>, we get.

$$\boxed{f(t) = -\frac{1}{2}e^{-t} + 3e^{-2t} - \frac{5}{2}e^{-3t}}$$

Example for You. ?

Example - 3 calculate I.L.T. for  $F(s) = \frac{4}{s(s+1)(s+4)}$

Solu<sup>n</sup>  $A = 1, B = -\frac{4}{3}, C = \frac{1}{3}$ .

$$\boxed{f(t) = 1 - \frac{4}{3}e^{-t} + \frac{1}{3}e^{-4t}}$$

Example - 4 calculate I.L.T. for  $F(s) = \frac{12}{(s+2)^2(s+4)}$ .

Solu<sup>n</sup>  $A = 6, B = -3, C = 3$ .

$$B = \left. \frac{d}{ds} \left[ (s+2)^2 \times \frac{12}{(s+2)^2(s+4)} \right] \right|_{s=-2}$$

$$\Rightarrow B = -3$$

$$\boxed{f(t) = 6te^{-2t} - 3e^{-2t} + 3e^{-4t}}$$

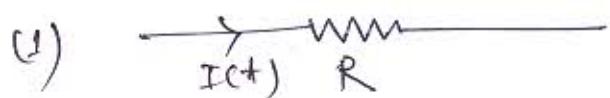
TRANSFER FUNCTION REPRESENTATION  $\Rightarrow$ 

Transfer function  $\Rightarrow$  it is defined as the ratio of Laplace transform of output signal or response to the Laplace transform of input signal or response taking all initial conditions are zero.

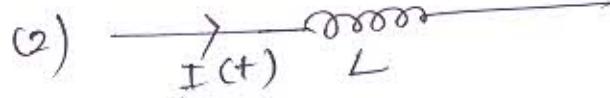
$$\boxed{T.f = G(s) = \frac{V_2(s)}{V_1(s)}}$$

$$Z(s) = \frac{V(s)}{I(s)} \rightarrow \text{impedance function.}$$

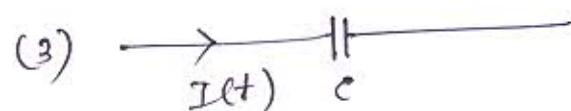
$$Y(s) = \frac{I(s)}{V(s)} \rightarrow \text{Admittance function.}$$



$$\begin{matrix} \text{in time domain} \\ V(t) = RI(t) \end{matrix} \rightarrow \begin{matrix} \text{in s-domain} \\ V(s) = RI(s) \end{matrix}$$

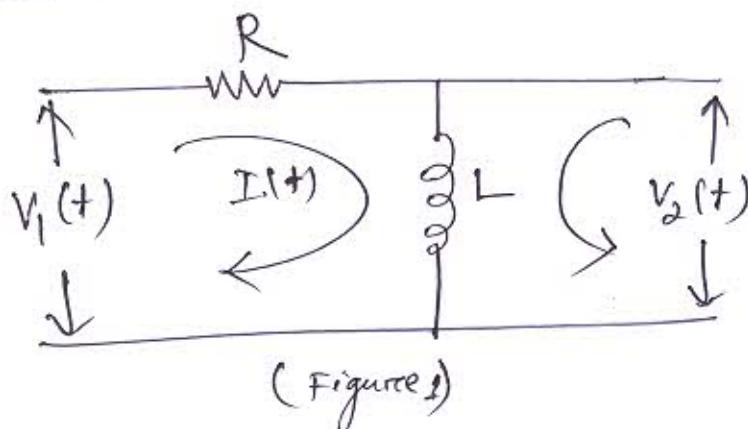


$$V(t) = L \frac{dI(t)}{dt} \rightarrow V(s) = LS I(s)$$



$$V(t) = \frac{1}{c} \int I(t) dt \rightarrow V(s) = \frac{1}{cs} I(s)$$

Example - (1.) calculate transfer function  $\frac{V_2(s)}{V_1(s)}$  in the given network.



Solution. Applying KVL in input loop.

$$V_1(t) = RI(t) + L \frac{dI(t)}{dt} \quad (1)$$

Applying KVL in output loop

$$V_2(t) = L \frac{dI(t)}{dt} \quad (2)$$

Apply Laplace Transform in equ<sup>n</sup> (1) and (2)  
we get.

$$V_1(s) = RI(s) + LS I(s)$$

$$\Rightarrow V_1(s) = I(s) [R + LS] \quad (3)$$

$$V_2(s) = LS I(s) \quad (4)$$

So, Transfer function.

$$T-f = h(s) = \frac{V_2(s)}{V_1(s)} = \frac{LS I(s)}{[R + LS] I(s)}$$

$$\boxed{h(s) = \frac{LS}{R + LS}}$$

Example - 2 Calculate Transfer function. in the given figure.

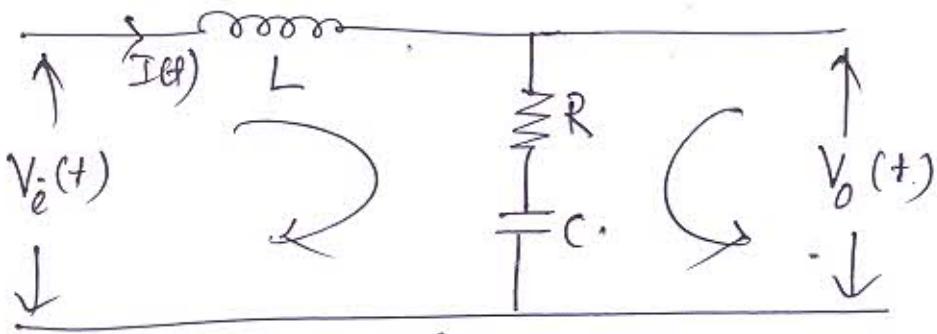


Figure - (2)

Solution Applying KVL in input loop.

$$V_i(t) = L \frac{dI(t)}{dt} + RI(t) + \frac{1}{C} \int I(t) dt \quad \textcircled{1}$$

Applying KVL in output loop.

$$V_o(t) = RI(t) + \frac{1}{C} \int I(t) dt \quad \textcircled{2}$$

Taking L.T. of above eqn. we get.

$$\Rightarrow V_i(s) = LS I(s) + RI(s) + \frac{1}{Cs} I(s)$$

$$\Rightarrow V_i(s) = I(s) [R + LS + \frac{1}{Cs}] \quad \textcircled{3}$$

$$\Rightarrow V_o(s) = RI(s) + \frac{1}{Cs} I(s)$$

$$\Rightarrow V_o(s) = I(s) \left[ R + \frac{1}{Cs} \right] \quad \textcircled{4}$$

$$\text{Transfer function} = G(s) = \frac{V_o(s)}{V_i(s)}$$

$$\Rightarrow \frac{V_o(s)}{V_i(s)} = \frac{jes \left[ R + \frac{1}{es} \right]}{jes \left[ R + LS + \frac{1}{es} \right]}$$

$$= \frac{\frac{Rcs + 1}{es}}{\frac{Rcs + Lcs^2 + 1}{cs}}$$

$$G(s) = \frac{V_o(s)}{V_i(s)} = \frac{1 + RCS}{1 + RCS + Lcs^2}$$

Initial Value Theorem :→

With the help of this theorem we can find the initial value of the time function  $f(t)$ . Thus if  $F(s)$  is the Laplace transform of  $f(t)$ , then according to this theorem,

$$f(0^+) = \lim_{t \rightarrow 0^+} f(t) = \lim_{s \rightarrow \infty} s \cdot F(s)$$

→ The only restriction is that  $f(t)$  must be continuous or at most, a step discontinuity at  $t=0$ .

Final Value Theorem :→

With the help of this theorem we can find the final value of the time function  $f(t)$ . Thus if  $F(s)$  is the Laplace Transform of  $f(t)$ , then according to this theorem

$$f(\infty) = \lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} s F(s)$$

→ The only restriction is that the roots of the denominator polynomial of  $F(s)$ , i.e. poles of  $F(s)$  have negative or zero real parts.

Poles and zeros :→

Consider the equation.

$$\frac{C(s)}{R(s)} = G(s) = \frac{K(s+z_1)(s+z_2)(as^2+bs+c)}{(s+p_1)(s+p_2)(As^2+Bs+C)} \quad \text{--- (1)}$$

where  $K = \frac{bm}{an}$  is known as the gain factor,  $s'$  is the complex frequency.

Poles :→ The poles of  $G(s)$  are those values of  $s'$  which make  $G(s)$  tend to infinity. For example in equn(1), we have poles at  $s = -p_1, s = -p_2$  and a pair of poles at

$$s = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

Zeros :→ The zeros of  $G(s)$  are those values of  $s'$  which make  $G(s)$  tend to zero. For example in equn(1), we have zeros at  $s_1 = -z_1, s_2 = -z_2$  and a pair of zero's at

$$s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

→ If either poles or zeros coincide, then such type of poles or zeros are called multiple poles or multiple zeros, otherwise they are known as simple poles or simple zero's.

Example (1) Determine poles and zeros of given function.

$$F(s) = \frac{s(s+1)}{(s+2)(s+4)}$$

Solu<sup>n</sup> in given function.

Zeros  $\Rightarrow s = 0, s = -1$ .

Poles  $\Rightarrow s = -2, s = -4$ .

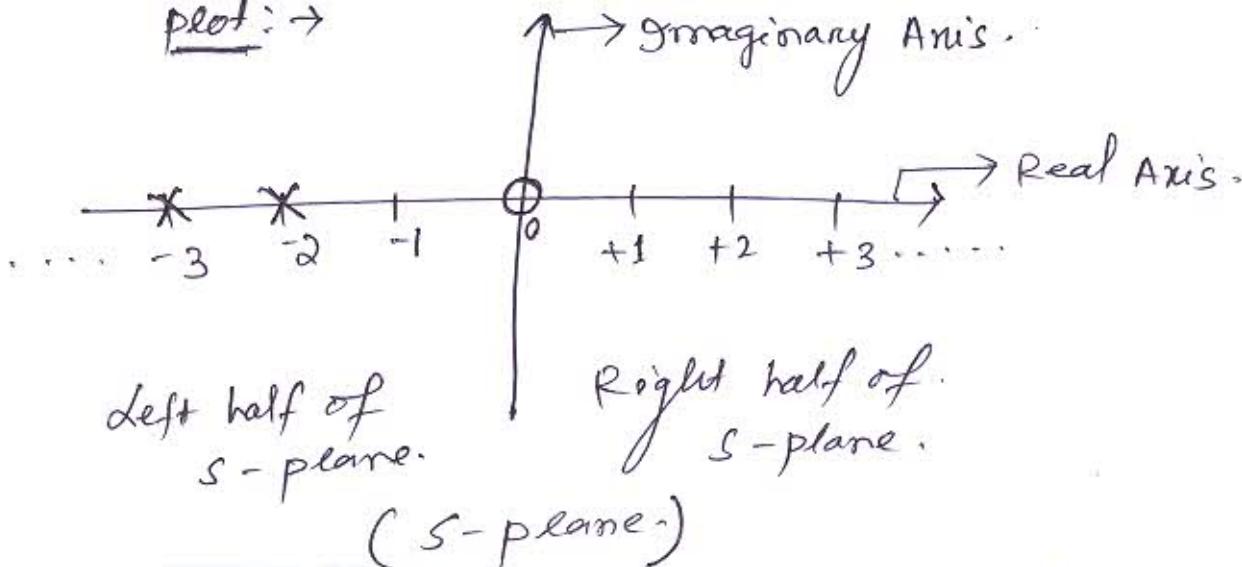
Example - 2 Draw the pole zero plot for given function.

$$F(s) = \frac{s}{(sf_2)(s+3)}$$

Solu<sup>n</sup> zeros  $s=0$

poles  $s=-2, s=-3$ .

plot :  $\rightarrow$



### Frequency Response (magnitude and phase plots)

→ The magnitude function and the phase function are the two plots in the frequency response characteristics and  $\omega$  is the common variable in between them.

→ Definition of Frequency response : → It is the steady state response of a system to a sinusoidal input of frequency  $\omega$  ( $0 < \omega < \infty$ ). The amount of amplification together with the phase shift are referred to as the frequency response data.

## Frequency Response specifications.

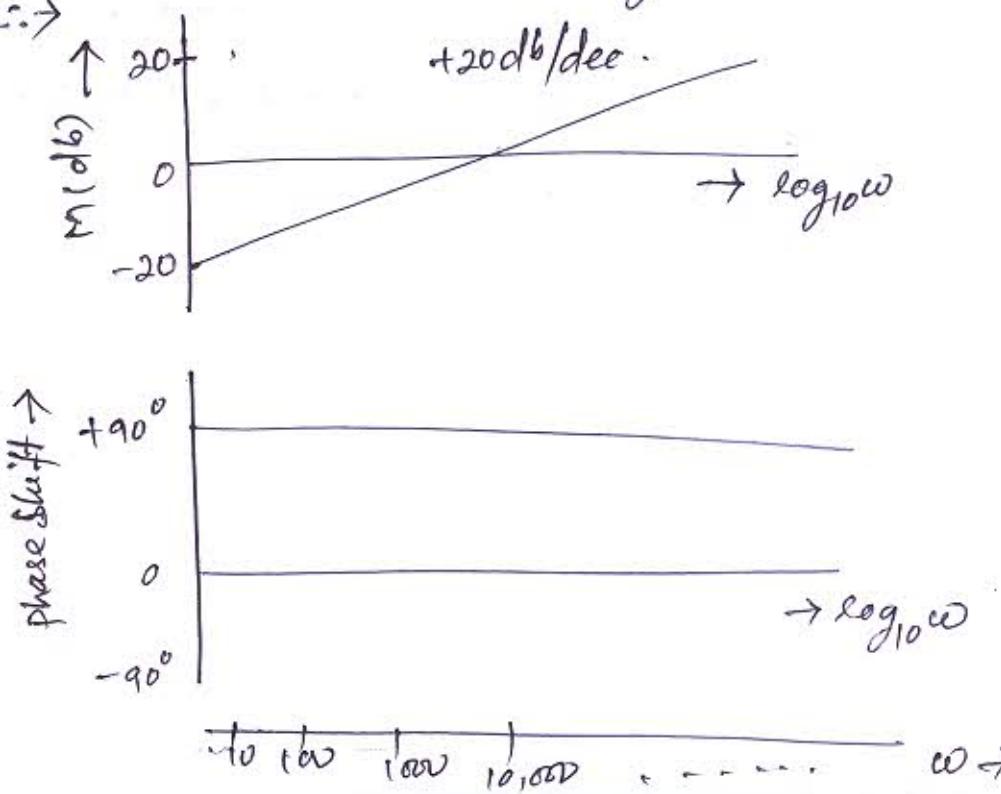
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- resonant frequency ( $\omega_r$ ) → cut off rate'
- Resonant peak ( $M_r$ ) → phase Margin (PM)
- Band width ( $\omega_b$ ) → gain Margin (GM)
- cut off frequency ( $\omega_c$ )

Bode plot :- Bode plot is a graphical representation of the transfer function for determining the stability of the control system. Bode plot consists of two separate plots. one is a plot of the logarithm of the magnitude of a sinusoidal transfer function, the other is a plot of the phase angle, both plots are plotted against the frequency.

- The curves are drawn on semilog graph paper, using the log scale for frequency and linear scale for magnitude (in decibels) or phase ~~angle~~ (in degrees).
- The magnitude is represented in decibels.
- Bode plot consists of
  - (i)  $20 \log_{10} |G(j\omega)|$  vs  $\log \omega$
  - (ii) phase shift vs  $\log \omega$ .

Example :-



Solution :-

$$V_{\text{Total}} = V_{L_1} + V_{L_2} + V_{L_3} + V_{L_4}$$

$$V_{L_1} = L_1 \frac{dI}{dt} + M_{12} \frac{dI}{dt} - M_{14} \frac{dI}{dt} = (L_1 + M_{12} - M_{14}) \frac{dI}{dt}$$

$$V_{L_2} = (L_2 + M_{12} + M_{23}) \frac{dI}{dt}$$

$$V_{L_3} = (L_3 + M_{23} - M_{34}) \frac{dI}{dt}$$

$$V_{L_4} = (L_4 - M_{34} - M_{14}) \frac{dI}{dt}$$

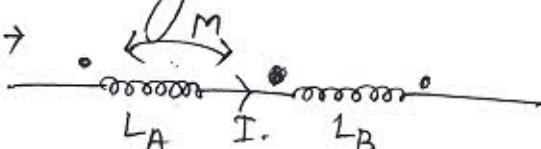
$$V_L = (L_1 + M_{12} - M_{14} + L_2 + M_{12} + M_{23} + L_3 + M_{23} - M_{34} + L_4 - M_{34} - M_{14}) \frac{dI}{dt}$$

$$= 14 \frac{dI}{dt}$$

$$\Rightarrow L_{\text{eq}} \frac{dI}{dt} = 14 \frac{dI}{dt} \Rightarrow \boxed{L_{\text{eq}} = 14 \text{ H}}$$

Problem :- Find the expression for the mutual inductance in series connection of two coupled coils, when the flux of the two coils assist each other, the net equivalent inductance being  $L_1$  and  $L_2$  for opposing of fluxes.

Solution :-



$$\text{Net inductance } L_1 = L_A + L_B + 2M \quad \text{--- (1)}$$

Similarly with opposition of fluxes in the two coils, the inductance ( $L_2$ ) =  $L_A + L_B - 2M \quad \text{--- (2)}$

Subtracting equation (2) from (1)

$$L_1 - L_2 = 2M - (-2M) = 4M$$

$$\Rightarrow \boxed{M = \frac{L_1 - L_2}{4}}$$

Dept - 10

## Resonance

06/05/2020

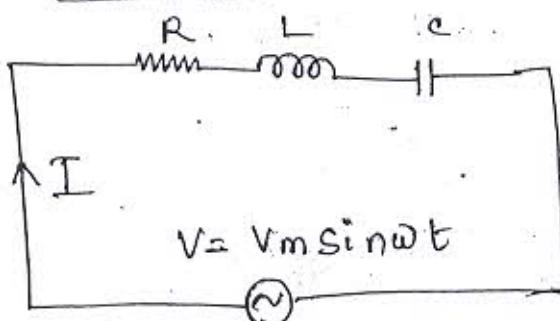
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- Resonance in electrical circuits consisting of passive and active elements represents a particular state of the circuit when the current or voltage in the circuit is maximum (or) minimum current the magnitude of excitation at a particular frequency, the circuit impedance being either minimum or maximum at the power factor unity.
- Depending upon the arrangement of passive elements in the circuit, resonance is of two types.  
(1) series resonance (2) parallel resonance.

RESONANCE

It is of two types.

- (a) Series resonance circuit
- (b) Parallel resonance circuit.

Series resonance circuit

$$\begin{aligned} Z &= R + j(X_L - X_C) \\ &= R + j(\omega L - \frac{1}{\omega C}) \end{aligned}$$

$$\omega L - \frac{1}{\omega C} = 0$$

$$\omega L = \frac{1}{\omega C}$$

$$2\pi f L = \frac{1}{2\pi f C}$$

$$\Rightarrow f^2 = \frac{1}{4\pi^2 LC}$$

$$\Rightarrow f_0 = \frac{1}{2\pi \sqrt{LC}}$$

This is called resonance frequency.

## Power factor :-

$$\cos \phi = \frac{R}{Z}$$

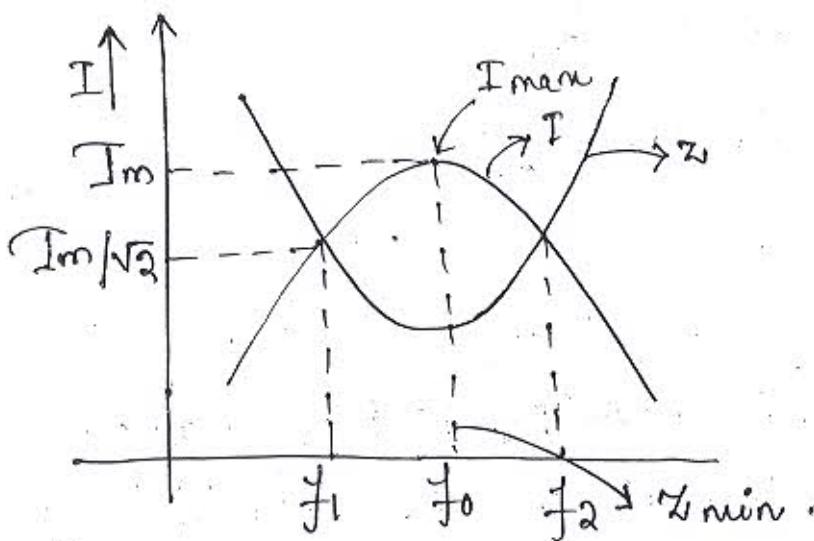
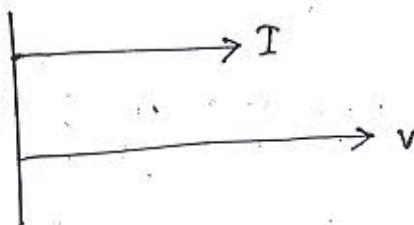
$$Z = R \text{ (At } X=0) = \frac{R}{R} = 1 \text{ (unity)}$$

$$\boxed{I = \frac{V}{Z}}$$

$X$  = minimum

$I$  = maximum under resonance condition

## Phase diagram :-



$f_1$  and  $f_2 \rightarrow$  Lower and upper half-freq.  
 $\rightarrow$  Lower and upper 3dB freq.

$$\boxed{B.W = f_2 - f_1}$$

### Properties of Series Resonance circuit :-

- The circuit purely resistive.  
(Total reactance ( $\omega$ ) = 0)
- The power factor is unity.
- The current (I) is maximum and impedance ( $Z$ ) is minimum.
- The voltage and I lying in same phase.
- Resonance frequency  $f_0 = \frac{1}{2\pi\sqrt{LC}}$

$f_1 = f_0 \left(1 - \frac{1}{2Q_0}\right)$	$BD = f_2 - f_1$
$f_2 = f_0 \left(1 + \frac{1}{2Q_0}\right)$	

### Quality factor ( $Q_0$ ):-

It is defined as the ratio of voltage across the inductor or capacitor to the applied voltage : It is denoted by  $Q_0$ :

$$Q_0 = \frac{V_L}{V} \quad Q_0 = \frac{V_C}{V}$$

$V_L$  and  $V_C$  → voltage across inductor and capacitor.

$$= \frac{I_0 X_L}{I_0 R} = \frac{X_L}{R} = \frac{I_0 X_C}{I_0 R} = \frac{X_C}{R}$$

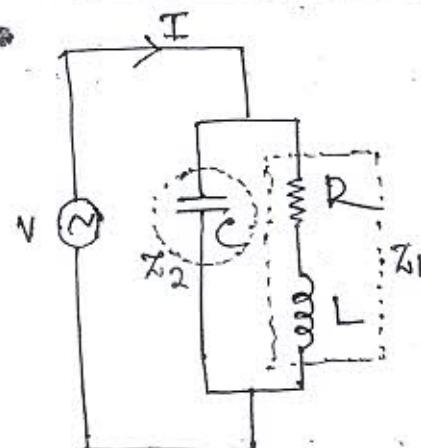
$$Q_0 = \frac{\omega_0 L}{R}$$

$$Q_0 = \frac{1}{\omega_0 C R}$$

Selectivity :-

It is defined as the ratio of resonance frequency to quality factor  $\frac{f_0}{Q_0}$ .

Parallel Resonance circuit :-



Impedance,  $Z_1 = R + j\omega L$

Admittance,  $y_1 = \frac{1}{\text{impedance}(Z_1)}$

$$= \frac{1}{R + j\omega L}$$

$$= \frac{R - j\omega L}{R^2 + \omega^2 L^2}$$

$$y_2 = j\omega C \quad (Z_2 = \frac{1}{j\omega C})$$

$$y = y_1 + y_2$$

$$= \frac{1}{R + j\omega L} + j\omega C$$

$$= \frac{R - j\omega L}{(R + j\omega L)(R - j\omega L)} + j\omega C$$

$$= \frac{R - j\omega L}{R^2 + \omega^2 L^2} + j\omega C$$

$$= \frac{R}{R^2 + \omega^2 L^2} + j\left(\omega C - \frac{\omega L}{R^2 + \omega^2 L^2}\right)$$

At resonance  $\Rightarrow$  (the total reactance = 0)

$$X_C = \frac{\omega L}{R^2 + \omega^2 L^2} = 0$$

$$\omega C = \frac{\omega L}{R^2 + \omega^2 L^2}$$

$$\boxed{R^2 + \omega^2 L^2 = \frac{L}{C}} \quad \cdots \cdots \text{(A)}$$

$$\omega^2 L^2 = \frac{L}{C} = R^2$$

$$\omega^2 = \frac{1}{LC} - \frac{R^2}{Z^2}$$

$$4\pi^2 f^2 = \frac{1}{LC} - \frac{R^2}{L^2}$$

$$f^2 = \frac{1}{4\pi^2} \left[ \frac{1}{LC} - \frac{R^2}{L^2} \right]$$

$$\boxed{f_0 = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}}$$

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \sqrt{1 - \frac{CR^2}{L}}$$

$$Q_0 = \frac{\omega_0 L}{R} \Rightarrow Q_0 = \frac{1}{\omega_0 CR}$$

$$Q_0^2 = \frac{\omega_0 L}{R} \times \frac{1}{\omega_0 CR} = \frac{1}{CR^2}$$

$$\frac{1}{Q_0^2} = \frac{CR^2}{L} \text{ putting in equation (1)}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \sqrt{1 - \frac{1}{Q_0^2}}$$

$$f_0 (\text{parallel}) = f_0 (\text{series}) \sqrt{1 - \frac{1}{Q_0^2}}$$

Calculation of Impedance :-

$$Y = Y_1 + Y_2$$

$$= \frac{1}{R + j\omega L} + j\omega C$$

$$= \frac{R}{R^2 + \omega^2 L^2} + j\left(\omega C - \frac{\omega L}{R^2 + \omega^2 L^2}\right)$$

At resonance:  $X = 0$

$$Y = \frac{R}{R^2 + \omega^2 L^2} \quad \dots \dots (2) \quad \left[ R^2 + \omega_0^2 L^2 = \frac{L}{C} \rightarrow A \right]$$

Putting the value of equation (1) in (2)

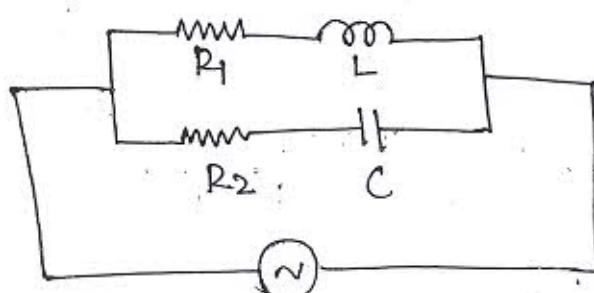
$$Y = \frac{R}{(L/C)} = \frac{1}{(L/CR)} = \frac{CR}{L}$$

$$Z = \frac{L}{CR} = R_a \quad \text{dynamic resistance}$$

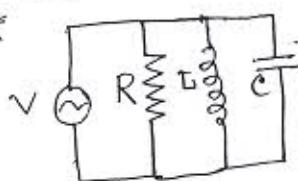
The impedance is purely Resistive and is called dynamic Resistance of the parallel resonance circuit.

Properties of parallel resonance circuit:

- Power factor of circuit is unity.
- Current at resonance is minimum and it is in the phase of applied voltage.
- Net impedance at resonance is maximum ( $L/CR$ )
- The admittance is minimum.
- $f_0 = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$  (Resonance frequency)



$$f_0 = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R_a^2}{L^2}}$$



$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

Sheet - II (Extra class) on 08/05/2020

Solved Examples :-

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Exa-1) What is the resonance frequency of a series RLC circuit where  $R = 10\Omega$ ,  $L = 25\text{mH}$  and  $C = 100\text{nF}$ ? Evaluate the Q factor also.

Solu<sup>n</sup>

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{25 \times 10^{-3} \times 100 \times 10^{-9}}} \text{ Hz}$$
$$= 100.71 \text{ Hz}$$

Q-factor  $\text{Q}_0 = \frac{\omega_0 L}{R}$  or  $\frac{1}{\omega_0 R C}$ .

using  $\text{Q} = \frac{\omega_0 L}{R} = \frac{2\pi \times 100.71 \times 25 \times 10^{-3}}{10}$

$$= 1.58$$

Exa-2) A 50  $\mu F$  capacitor, when connected in series with a coil having  $40\Omega$  resistance, resonates at 1000 Hz. Find the inductance of the coil. Also obtain the circuit current if the applied voltage is 100 V. Also calculate the voltage across the capacitor and the coil at resonance.

Solu<sup>n</sup> At resonance.

$$X_L = X_C \text{ or } 2\pi f_0 L = \frac{1}{2\pi f_0 C}$$

$$\Rightarrow L = \frac{1}{4\pi^2 f_0^2 C} \text{ H}$$

$$= \frac{1}{4\pi^2 \times (1000)^2 \times 50 \times 10^{-6}} = 0.5 \text{ mH}$$

$$|I_0| = \frac{V}{Z} = \frac{V}{R} \text{ at Resonance.}$$

$$= \frac{100}{40} = 2.5 \text{ A.}$$

$$[\text{power loss of the coil} = I_0^2 R = (2.5)^2 \times 40 = 250 \text{ watt}]$$

$$\text{Again } V_C = I_0 X_C = 2.5 \times \frac{1}{2\pi f_0 C}$$

$$= 2.5 \times \frac{1}{2\pi \times 1000 \times 50 \times 10^{-6}} = 7.96 \text{ V}$$

$$\text{Again } X_L = \omega L = 2\pi \times 1000 \times 0.5 \times 10^{-3}$$

$$= 3.14 \Omega$$

$$V_{\text{coil}} = I_0 Z_{\text{coil}} \text{ (at Resonance)}$$

$$= 2.5 \sqrt{(40)^2 + (3.14)^2} = 100.31 \text{ V}$$

Exa-3 A series circuit comprises an inductor of resistance 10 ohms and  $L = 159 \times 10^{-6} \text{ H}$  and a variable capacitor connected to a 50 volt sinusoidal supply of frequency 1 MHz. What value of capacitance will result in resonant conditions and what will then be the current? Calculate the Q-factor of the circuit.

Solu<sup>n</sup>  $R = 10 \Omega$ ,  $L = 159 \times 10^{-6} \text{ H}$ ,  $c = ?$

$$E = 50 \times 10^3 \text{ volt}, f = 10^6 \text{ Hz}$$

$$f = \frac{1}{2\pi\sqrt{LC}}$$

$$\Rightarrow c = \frac{1}{4\pi^2 f^2 L} = \frac{1}{4\pi^2 (10^6)^2 \times 159 \times 10^{-6}} = 159.47 \text{ PF}$$

$$\text{Current } I = \frac{50 \times 10^3}{10} = 5 \times 10^3 \text{ mA.}$$

Q factor of the circuit.

$$= \frac{1}{R} \sqrt{\frac{L}{c}} = \frac{1}{10} \sqrt{\frac{159 \times 10^{-6}}{159.47 \times 10^{-12}}} = 99.85$$